National University of Singapore
Department of Electrical and Computer Engineering
MCH5002 Applications of Mechatronics

MODELLING, SIMULATION AND CONTROL OF A DC MOTOR

1. OBJECTIVES

This is a hands-on session on the application of computer-based control to a voltage-controllable electro-mechanical system – the DC motor. In the session, the student will go through a complete control design cycle, including the modeling, simulation and control design of the system, fully instrumented with position and velocity measurements. National Instrument's LabVIEW will be the software platform used for the experiment. At the end of the experiment, the student should have some experience in

- Static and dynamic modelling of the DC motor
- Simulation-based model validation and control design
- Real-time feedback control of the system for velocity tracking

To benefit more fully from this session, students should read the manual and answer the pre-laboratory questions (Q1-Q3) before going to the laboratory.

2. A MATHEMATICAL MODEL FOR DC MOTOR - BACKGROUND

A mathematical relationship between the shaft angular velocity and voltage input to the DC motor may be derived from physical laws. The angular position may then be obtained from an integration of the angular velocity. With Newton's law of motion, we may obtain the following relationship:

$$J \frac{d\omega(t)}{dt} = \Gamma(t) \quad , \tag{1}$$

where $\omega(t)$ is the velocity in rad/sec at time t, J is the inertia of motor and its load, and $\Gamma(t)$ is the net torque given effectively by

$$\Gamma(t) = \tau_e(t) - \tau_L(t) - k_\omega \omega(t) \quad , \tag{2}$$

where k_ω is a damping coefficient, $\tau_L(t)$ is the load torque and $\tau_e(t)$ is the generated torque. If we further assume that the generated torque is proportional to the applied voltage, $v(t)$, we have the following differential equation describing the motor:

$$J \frac{d\omega(t)}{dt} = kv(t) - \tau_L(t) - k_\omega \omega(t) \quad . \tag{3}$$

The velocity can be measured with a tachogenerator, albeit in an electrical voltage form. We assume that the actual velocity is linearly related to the voltage output from the tachogenerator, y(volts).

$$\omega = my + c \tag{4}$$

The differential equation in terms of tachogenerator output voltage and the applied voltage input is thus

$$Jm\frac{dy(t)}{dt} = kv(t) - \tau_L(t) - k_\omega my(t) - k_\omega c \tag{5}$$

or equivalently

$$T\frac{dy(t)}{dt} + y(t) = Kv(t) - k_L \tau_L(t) - k_C \tag{6}$$

where the time constant $T = J / k_\omega$, the steady state gain $K = k / k_\omega m$ and the constants $k_L = 1 / k_\omega m$, $k_C = c / m$. This is a first order differential equation. If we further assume the load torque τ_L to be a constant, T, K, $k_L \tau_L$, k_c are the parameters of the DC motor model.

We shall regard $v(t)$, the applied voltage as the input to the system, $y(t)$ the measured output of the system.

Q1:
If both the input voltage and the load disturbance are constant, what is the steady–state tachogenerator voltage, i.e. y(t) as $t \to \infty$?

(HINT: For stable systems, $\frac{d}{dt} \to 0, t \to \infty$)

Q2:
A proportional controller of the form $v(t) = k_P(y_{SP} - y)$ is used on this plant. If y_{SP} is also constant, what is the steady-state tachogenerator voltage, i.e., $y(t)$ as $t \to \infty$? (HINT: For stable systems, $\frac{d}{dt} \to 0, t \to \infty$)

Q3:
With the proportional controller, what is the time constant of the closed loop system? How is the magnitude compared to the open-loop time constant? (HINT: Put in the form similar to (6) below (where coefficient of y=1)

$$T_{cl}\frac{dy}{dt} + y = ...$$

T_{cl} is the closed-loop time constant.

3. APPARATUS

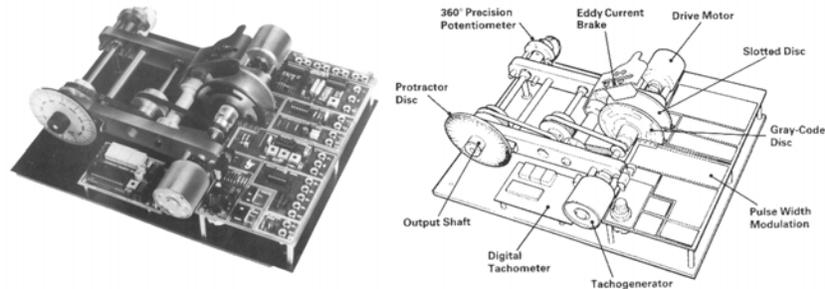- DC servo motor setup (Fig. 1)

Fig.1: DC Servo motor

- PC with installed data acquisition cards (2 channels of analog output and 8 channels of analog input), MATLAB and LabVIEW software
- Power supply for DC motor set
- Printer

4. INPUT-OUTPUT CONFIGURATION

The data acquisition box should be configured so that analog input channel 1 reads the motor output position voltage signal, analog input channel 2 reads the tachogenerator output voltage signal and analog output channel 0 drives the motor. These configurations have been done in advance.

5. INSPECTING THE SYSTEM

Identify the DC motor, the tachogenerator, the output shaft, the output position potentiometer, the load magnet arm; the RPM display indicator, and the angular position indicator. **Ensure that the load arm is down so that the maximum load torque is applied.**

6. STARTING LABVIEW

The experiment uses the software LabVIEW where various control configurations can be easily organised. LabVIEW, or Laboratory Virtual Instrument Engineering Workbench, is a graphical programming language that has been widely adopted throughout industry, academic, and research labs for data acquisition and instrument control software. It offers a flexible instrumentation and analysis software platform since computers are much more flexible than standard instruments.

Double click **MCH 5002.exe** on the desktop to retrieve the user interface preprogrammed for you to carry out the experiment. Fig. 2 is a snapshot of the screen you should see.
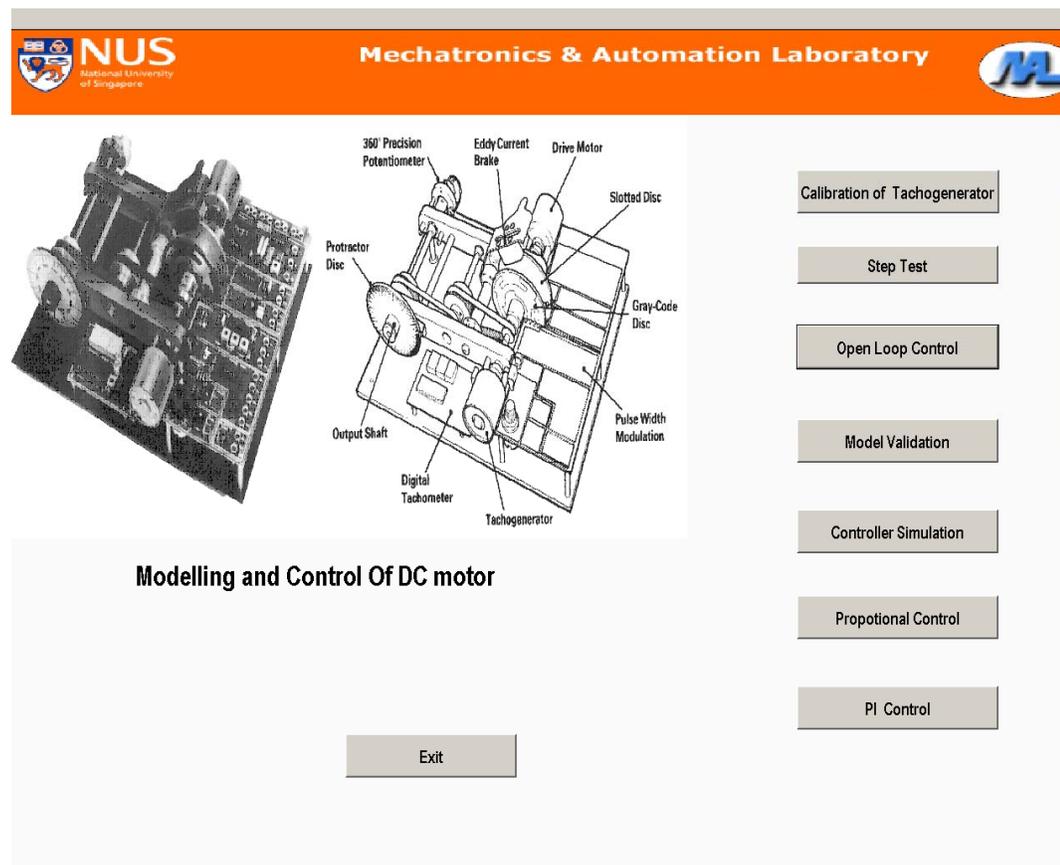

Fig. 2: User interface

7. PROCEDURES

7.1 Modelling of the DC Motor

The motor speed as measured by the tachogenerator output is the output of the system, and the applied voltage is the input to the system. This part of the experiment will let you obtain the model parameters (T, K, $k_L\tau_L$, k_c) in (6) of the system. The model allows us to perform simulation, control and their applications. We begin by finding out the relationship between motor speed measured in volts by the tachogenerator and the speed as displayed on the red digital display.

7.1.1 Calibrating the Tachogenerator

The motor velocity is measured with a tachogenerator. The voltage output from the tachogenerator is approximately linearly related to the velocity. In this part of the experiment, we will find this linear relationship.

Click on **Calibration of Tachogenerator** button in the user interface.

Set applied voltage output first to -5 volts. Allow a few seconds for the speed to settle down. In Table 1 below, record the tachogenerator output in volts as well as the displayed speed in rpm (define the sign of the speed on the RPM display indicator based on the direction of the rotation yourself). Complete Table 1 below with the other inputs.

Table 1 Calibration of Speed Sensor

Applied voltage	Tachogenerator (Volts) (y)	Velocity in rpm	Velocity in rad/sec (ω)
-5 volts			
-3 volts			
-1 volts			
0 volts			
1 volts			
3 volts			
5 volts			

Click on **Back** button to return to the main menu if the calibration is completed.

Q4:
Using the data in Table 1, plot a graph of the speed (ω) in rad/sec vs the tachogenerator output voltage (y). From the graph, estimate the coefficients m and c in Equation (4) (you may use MATLAB, EXCEL or other software). Thus, $k_c=c/m$ can be determined.

Q5:

Using the data in Table 1, plot a graph of the speed (ω) in rad/sec vs the applied voltage (v).

From (3), at steady state and assuming load torque is constant, we have

$$\omega = \frac{k}{k_\omega} v - \frac{1}{k_\omega} \tau_L$$

From the plot and knowing m, we can therefore obtain K=k/ ($k_\omega m$) and $k_L \tau_L$ where k_L=1/(k_ωm).

Comment on the linearity of this relationship, giving possible reasons for nonlinearities, if any. The only parameter left to be estimated is the time constant *T* which we will obtain next.

7.1.2 Estimating K and T from Step Tests

- Click on the **Step Test** button to activate the program to apply a square wave (a series of steps). The magnitude and bias of the square wave is set by corresponding number controls.
- Using the number controls, set the bias to 1 volt and the magnitude to 1.5 volts. The applied voltage is then a square wave with voltages 1 volt and 2.5 volts. Observe the step response on the chart. Save the response for printing. (click on pause button→right click on the plot →export simplified image→ save at desktop).

Q6:

From the chart, we can estimate the time constant *T* in Equation (6). From the same chart, we can also obtain *K* to compare with what is obtained earlier in 7.1.1. Let's see how to obtain these parameters.

Assuming the system is in a steady state with v_0 and y_0, related by:

$$y_0 = Kv_0 - k_L \tau_L - k_C$$

Then let v changes to $v_0+\Delta v$, and y becomes $y_0+\Delta y(t)$. Substituting into (6), we have

$$T \frac{d(y_0 + \Delta y(t))}{dt} + y_0 + \Delta y(t) = Kv_0 + K\Delta v - k_L \tau_L - k_C$$

Since y_0 is a constant, $dy_0/dt=0$ and this equation simplifies to

$$T\frac{d\Delta y(t)}{dt} + \Delta y(t) = K\Delta v$$

The solution to this differential equation is:
$$\Delta y(t) = K(1 - e^{-t/T})\Delta v.$$

Put t=T,

$$\Delta y(T) = 0.63K\Delta v = 0.63\Delta y_{ss},$$

where Δy_{ss} is the steady state change in y. i.e. $\Delta y_{ss} = \Delta y(t \to \infty) = y(t \to \infty) - y_0$.

Therefore, we can obtain T from the chart as the time for y to reach 63% of its final value. K can also be obtained from

$$K = \frac{\Delta y_{ss}}{\Delta v}.$$

7.2 Model Validation via Simulation

After a model is obtained, it should be validated to assess if it adequately describe the actual system. In this part, we will compare the response of the system and the model to the same input, and observe the difference.

Click on **Model Validation** button in the user interface. Enter the model parameters thus obtained and begin the step input to the model. Log the step response and compare it to the earlier step response obtained in 7.1.2,

Repeat 7.1 if the model obtained is not adequate.

7.3 Velocity Control

We will experiment with both the manual (open-loop) control and automatic (closed-loop) control of the motor velocity.

7.3.1 Open-Loop (Manual) Control

Now, we would like the motor speed to track the set point (desired speed). We would also like to achieve this tracking of the set point even if there are plant load changes or disturbances. An example of a load disturbance is when we change the load torque by moving the load arm. Alternatively, you can try to hold the output disk with your fingers. Naturally, we also need a stable response.

In this part of the experiment, we examine the performance of open loop control or manual control.

- Click on the **Open Loop Control** button.
- Based on the static equation $\omega = \dfrac{k}{k_\omega} v - \dfrac{1}{k_\omega} \tau_L$, calculate the voltage you need so that the motor velocity is 100 rpm. Set the applied voltage to your estimated value.
- Now raise (or lower) the load arm. Observe carefully the measured speed. Save the plot of the chart (click on pause button→right click on the plot →export simplified image→ save at desktop)
- Now try to adjust the applied voltage to get the speed back to 100 rpm.

Q7:
Comment on the performance of open loop control. Consider the steady state error and the response to load disturbance.

7.3.2 Feedback Control Design Via Simulation

(i) Proportional control (P) only

Click on the **Controller Simulation** button and select controller type as **Proportional control**. Enter the model parameters you have obtained. The set point is a square wave. The controller is a proportional only controller. The control output is proportional to the resulting error signal. Thus, $v(t) = k_P[y_{SP} - y]$ where y_{SP} is the set point voltage (desired speed in terms of voltage).

Tune the P control gain to obtain a response which has a closed-loop time constant of not more than 0.3s. Record this gain for use when you work on the control implementation later in 7.3.3.

(ii) Proportional integral (PI) control

With proportional control, it was not possible to remove steady state error. In proportional plus integral control, the applied voltage is proportional to the sum of the error and its integral with respect to time. This means that the control signal will increase or decrease until the error is zero

$$v(t) = k_P\left(e(t) + \frac{1}{T_i} \int e(t)dt \right), \qquad e(t) = y_{SP} - y(t)$$

K_p is the proportional gain and T_i is the integral (reset) time constant.

Select controller type as **PI control**. The set point is a square wave too. Tune the proportional gain and the integral time constant to obtain a response which has not more than 5% overshoot, a closed-loop time constant of not more than 0.3s and zero steady state error. Record K_p and T_i for use when you work on the control implementation later in 7.3.3.

7.3.3 Real-time Feedback Control

(i) Proportional control (P) only

- Click on the **Proportional Control** button. The set point y_{SP} is a square wave with a bias voltage of 1 volt and an amplitude specified by the control magnitude. Begin with a small value of proportional gain.
- Save the transient and steady state response, and print it out later for your report. Note the steady state error.
- Now increase the proportional gain until the speed is unstable. Note the gain value which just causes instability. Obtain a chart of the unstable response at this point.
- Set the proportional gain to the value you obtained from the simulation in 7.3.2. Are you able to achieve the desired step response of a closed-loop time constant of not more than 0.3s? Otherwise, fine tune the P gain till the requirement is met. Save and print this chart.
- Now set the magnitude to 0 so that the set point is constant. Apply a load disturbance using the load arm. On one chart, obtain a record of the disturbance response.

Q8:
Use your experimental data to verify that the steady state error decreases as the proportional gain increases. From your experimental data, describe what happens to the transient response as the proportional gain becomes large? Without going into the mathematics, is this consistent with what you would expect from theory?

Q9:
Comment on the effect of the load disturbance.

Q10:
Summarise the conclusions of your experiment in terms of the effect of proportional feedback control on (a) the steady state error to set point change (b) the transient response to set point change (c) the steady state error to disturbance (d) the transient response to disturbance. (e) what you understand by an unstable feedback system.

(ii) Proportional plus integral (PI) control

- Click on the **PI Control** button.
- Using the number controls, set the proportional gain and integral time constant according to what was obtained from the simulation in 7.3.2. Are you able to achieve the specifications in terms of an overshoot of less than 5%, a closed-loop time constant better than 0.3s and zero steady state error to a step change in set point? Otherwise, fine tune the gains till the requirement is met. Record the transient and steady state response. Save and print this chart.
- Keep the proportional gain unchanged and reduce the integral (reset) time gradually. Record the transient and steady state response. Raise or lower the load arm and record the transient response to a load disturbance.

Q11:
Demonstrate with your experimental data that with any positive value of integral (reset) time, the steady state error to a set point change is zero.

Q12:
Comment on the transient response (both to set point change and to disturbance) when the integral (reset) time is reduced.

8. REPORT

The report should log the results from the experiment with your interpretations, observations and conclusions. You should try to answer all questions in the manual.